Geometric depolarization in patterns formed by backscattered light

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We formulate a framework to extend the idea of Berry's topological phase to multiple light scattering, and in particular to backscattering of linearly polarized light. We show that the randomization of the geometric Berry's phases in the medium leads to a loss of the polarization degree of the light, i.e., to a depolarization. We use Monte Carlo simulations in which Berry's phase is calculated for each photon path. Then we average over the distribution of the geometric phases to calculate the form of the patterns, which we compare with experimental patterns formed by backscattered light between crossed or parallel polarizers.

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The transport of light through human tissues is one of the most promising techniques to detect breast cancer, for instance, in a noninvasive way. For medical imaging applications, it is important to extract the information contained not only in the intensity but also in the polarization of backscattered light. This extraction is not easy in general because of the complexity of vector-wave multiple scattering. In this Letter we study a simple experiment, in which polarized light is backscattered from a diffuse medium. In these conditions a fourfold symmetry pattern can be observed between crossed polarizers that was first interpreted qualitatively by Dogariu and Asakura.[1] Recently more quantitative approaches were developed by use of Mueller matrices.[2,3] In this Letter we propose an alternate approach, which is quite simple to implement because it is not based on a vector radiative-transfer method as generally used in the literature. Instead our approach is based on the notion of geometric phase, which was introduced by Berry[4] in his interpretation of experiments showing optical activity in a helically wound optical fiber.[5] Berry's geometric phase in these references is the phase acquired by light when its direction of propagation is slowly changed on a sphere of directions (i.e., in momentum space). This geometric phase is equal to the solid angle on the sphere of wave-vector directions. A different geometric phase, called Pancharatnam's phase is the phase acquired by paraxial polarized light wave when its polarization undergoes some transformation on the Poincaré sphere. That geometric phase is equal to half the solid angle on the Poincaré sphere.[6] So far the applications of geometric phases to polarized light have been limited to situations in which light is traveling in a homogeneous medium. In this Letter we use only the first geometric phase, Berry's phase, and apply it to multiple light scattering in a random medium. Before presenting our application of Berry's phase, which follows closely and extends the recent Ref. 7, we discuss the cross-shaped patterns, using the standard Stokes formalism to make a connection with previous work.[2,3]

We assume that linearly polarized light is incident upon a medium and that the direction of the incident beam is normal. The intensity and polarization of the backscattered light are completely characterized by Stokes parameters (I, Q, U, V). Transformations of the Stokes parameters are represented by 4×4 Mueller matrices. Scattering matrix S is such a matrix and contains contributions from all orders of scattering.[3] The dependence of the outgoing Stokes parameters as a function of azimuthal angle ϕ measured about the incident beam direction can be obtained by a product of the appropriate Mueller matrices. We find that the outgoing intensities in the backscattered direction and between parallel (perpendicular) polarizers are[3]

$$I_{\perp} = \frac{1}{4}(2S_{11} + S_{33} - S_{22}) - \frac{1}{4}(S_{22} + S_{33})\cos 4\phi\,, \tag{1}$$

$$I_{\parallel} = \frac{1}{4}(2S_{11} - S_{33} + S_{22}) - S_{12} \times \cos 2\phi + \frac{1}{4}(S_{22} + S_{33})\cos 4\phi\,, \tag{2}$$

corresponding to Stokes parameters $I = I_{\perp} + I_{\parallel}$ and $Q = I_{\parallel} - I_{\perp}$. Equations (1) and (2) are valid for any distribution of randomly oriented particles with a symmetry plane. Note that Eq. (1) implies that the cross-polarized pattern has a fourfold symmetry, whereas Eq. (2) implies an additional twofold symmetry in the copolarized pattern because of the term proportional to S_{12}. In the particular case that is satisfied in multiple light scattering,[8] when $S = S_{11}\,\mathrm{diag}(1, C, C, D)$, with $C = S_{22}/S_{11}$ and $D = S_{44}/S_{11}$, Eqs. (1) and (2) take the simple form

$$I_\perp = \frac{1}{2} I_0(1 - C\cos 4\phi), \tag{3}$$

$$I_\parallel = \frac{1}{2} I_0(1 + C\cos 4\phi), \tag{4}$$

corresponding to outgoing Stokes parameters $I = I_0 = S_{11}$ and $Q = CI_0 \cos 4\phi$. Note that a cross is expected now in both polarization channels and that C measures the contrast of this pattern.

Let us now discuss the origin of the depolarization of polarized light. For Rayleigh scattering, the (linear) polarization vector after scattering, $\mathbf{E}'$, is $\mathbf{E}' = \mathbf{k}' \times (\mathbf{E} \times \mathbf{k}')$, in terms of the polarization vector before scattering $\mathbf{E}$ and the scattered wave vector $\mathbf{k}'$. This implies that $\mathbf{E}$ evolves by parallel transport in the limit of small scattering angles and diffuses on the sphere of wave-vector directions until the memory of the polarization has been lost. This depolarization has a characteristic length l_p equal to $2.8l$, where l is the elastic mean free path of the light.[9] As the anisotropy in the scattering increases, l_p approaches the transport mean free path l^*.[10,11] Here we assume forward-peaked scattering because it applies to many biological tissues, and because in this case there is a clear analogy between Berry's geometrical phase in optics and the twist and writhe of polymers.[7] The hypothesis of forward-peaked scattering nicely satisfies the requirement for the Berry's phase of a slow variation of the circuit in momentum space[6] and corresponds to a special recently investigated limit of the radiative-transfer equation.[12]

Let us consider a path of light, which we assume to be normally incident on a semi-infinite random medium. Following Ref. 4, we express polarization vector $\mathbf{E}$ in a basis of two vectors ($\mathbf{n}$, $\mathbf{b}$) normal to tangent vector $\mathbf{u}$ (if the path is regular enough, the Frénet frame is a possible choice), as shown in Fig. 1:

$$\mathbf{E}(t) = c_1(t)\mathbf{n}(t) + c_2(t)\mathbf{b}(t), \tag{5}$$

where t is a parameter that goes from 0 to s along the path. Let us call ϕ the angle between $\mathbf{E}$ and $\mathbf{n}$ at $t = 0$, so $c_2(0)/c_1(0) = \tan\phi$. Since the polarization evolves by parallel transport, $\dot{c}_1 = \tau c_2$ and $\dot{c}_2 = -\tau c_1$, where τ denotes the torsion on the trajectory, as found many years ago by Rytov.[13] In the backscattering geometry, $\mathbf{n}(t = s) = -\mathbf{n}(t = 0)$ and $\mathbf{b}(t = s) = \mathbf{b}(t = 0)$; therefore we find that the polarization vector at the end of the path is

$$\mathbf{E}(t = s) = -\cos[\phi + \Omega(s)]\mathbf{n}(s) + \sin[\phi + \Omega(s)]\mathbf{b}(s), \tag{6}$$

where $\Omega(s)$ is a geometrical phase equal to the opposite of the integral of the torsion between $t = 0$ and $t = s$ modulo 4π. In the analogy between a path of light and a semiflexible polymer, the twist of the path is zero for light (it would be nonzero only in a chiral medium), and the writhing angle is precisely Ω. This writhe is a real value since the path is open, and that real value is equal to the algebraic area of a random walk on a unit sphere, with the constraint that the path goes from the north pole to the south pole in the backscattering geometry. From Eq. (6), we find that the output intensity after the light has gone through an analyzer crossed with respect to the direction of the incident polarization is proportional to $\sin^2(2\phi + \Omega)$. Because the medium is random, this intensity must be averaged with respect to all paths:

$$I_\perp(R) = \int P'(s, R)\mathrm{d}s\langle \sin^2[2\phi + \Omega(s)]\rangle, \tag{7}$$

where $P'(s, R)$ is the distribution of the path length for a given distance to the incident beam R and $\langle\ldots\rangle$ denotes the average over paths of length s. Using the identity $2\sin^2(2\phi + \Omega) = 1 - \cos(4\phi)\cos(2\Omega) + \sin(4\phi)\sin(2\Omega)$ and the fact that $\langle\sin(2\Omega)\rangle = 0$, because the distribution of Ω is even, we write Eq. (7) in the form of Eq. (3) with $I_0(R) = \int P'(s, R)\mathrm{d}s$ and

$$C(R) = \frac{1}{I_0(R)} \int P'(s, R)\mathrm{d}s\langle \cos[2\Omega(s)]\rangle. \tag{8}$$

The factor $\cos(2\Omega)$ in Eq. (8) means that the contrast results from grouping pairs of paths of opposite geometrical phases, and the sum over s means that the phases of any other paths are uncorrelated. Interestingly, a similar randomization of the phase occurs in the theory of magnetoconductance of Anderson insulators.[14]

To evaluate the distributions of Ω for fixed s, $P(s, \Omega)$ shown in Fig. 2, we use a Monte Carlo algorithm originally developed for semiflexible polymers. Random paths are generated with an exponential distribution of path length with a characteristic step equal to l. The incident photons are normal to the interface, but when light is exiting the medium all

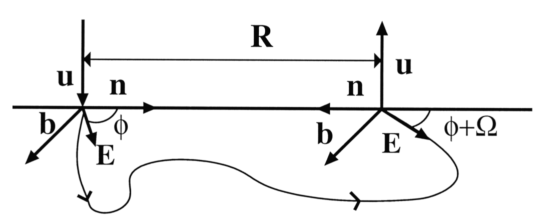

Fig. 1. Representation of a typical path in a semi-infinite random medium in backscattering. The Frénet frame consists of tangent $\mathbf{u}$, normal $\mathbf{n}$, and binormal $\mathbf{b}$ vectors. R denotes the distance between end points, ϕ is the initial angle between polarization vector $\mathbf{E}$ and normal $\mathbf{n}$, and Ω is the geometric phase.

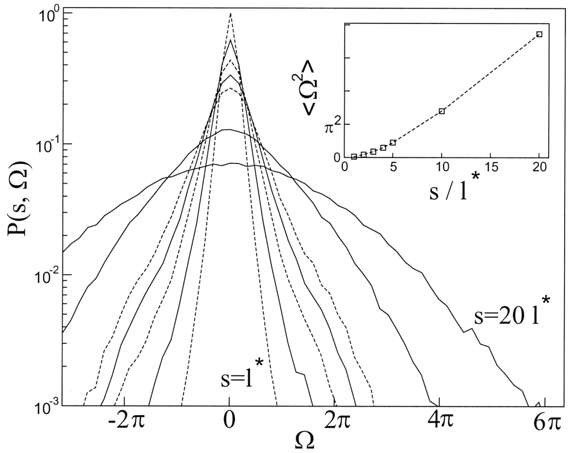

Fig. 2. Distribution of geometric phase Ω for different values of path length s and in the inset variance of the distribution as a function of s/l^*.

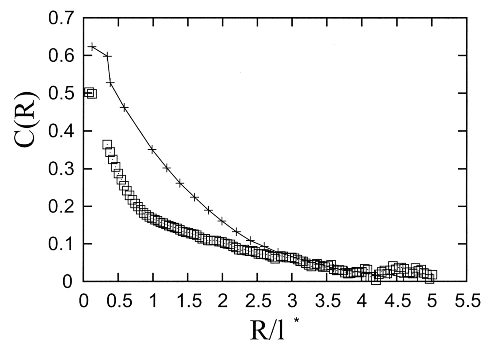

Fig. 3. Contrast as a function of R: the crosses were obtained from Monte Carlo simulations by use of Eq. (8), and the squares are experimental values, obtained from an analysis of Stokes parameter Q.

the outgoing angles of the emergent photons are accepted. The paths can be generated for an arbitrary ratio of l^*/l. We calculated the geometric phase by closing the paths on the momentum sphere with a geodesic.[7] Because of this closure the distribution of Ω for short paths, $s \ll l^*$, is peaked at zero, as is also found for planar random walks (Levy's law). For long paths, $s \gg l^*$, the distribution of Ω widens until the polarization is completely lost. In this regime the distribution $P(s, \Omega)$ should be Gaussian according to the central limit theorem. Indeed, we have confirmed this point by numerically evaluating the moment of order four of the distribution. Furthermore, the variance of the distribution, which was quadratic for $s \ll l^*$, becomes linear for $s \gg l^*$, as seen from the inset of Fig. 2. This means that $P(s, \Omega) = \sqrt{l_p/\pi s}\, \exp(-\Omega^2 l_p/s)$, which implies that $\langle \cos 2\Omega(s) \rangle = \exp(-s/l_p)$. In Fig. 3, we show the corresponding curve for the contrast of the pattern calculated from Eq. (8), together with experimental points, which we obtained by averaging Stokes parameter Q of an image along two perpendicular directions, thereby suppressing a possible contribution in $\cos(2\phi)$ present in Eq. (2). In the experiment a colloidal suspension of latex particles of negligible absorption (diameter 0.5 μm, wavelength $\lambda = 670$ nm) was used, and the sample was $\sim 8.8 l^*$ thick. The value of anisotropy parameter g in the simulation was chosen to match the experimental value $g \simeq 0.82$. In this figure one can see that the contrast decreases exponentially as a function of distance R with a characteristic distance of the order of $l_p \simeq l^*$, which agrees with both theory and experiments.[10,11] In the central region of the pattern, low-order scattering is dominant, as was confirmed numerically. This could explain the discrepancy between experiments and simulations in this region, since our model only treats low-order scattering events in an approximative way.

To conclude, we have developed a simple theoretical framework to extend the idea of Berry's topological phase to the backscattering of light in a multiple scattering medium. The randomization of the geometric phases is the process that leads to depolarization, which is most clearly seen when the scattering is peaked in the forward direction. We have substantiated our theory with experiments. We hope that our work will motivate further studies on the role of geometric phases in the transport properties of polarization in random media.

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