Finite size scaling in first and second order phase transitions

Références

[1] Principles of condensed matter physics, P. M. Chaikin and T. C. Lubensky, Cambridge Univ. Press

1 Critical exponents in mean-field theory

In Landau theory, the density of free energy for a magnetic system in a magnetic field h takes the form

$$f(m) = f_0 + \frac{r}{2}m^2 + um^4 - mh,$$

where m denotes the magnetization, r is of the form $a(T - T_c)$, and u is another coefficient independent of temperature. The magnetization is first assumed to be uniform spatially and the system is of infinite size. Such an expansion is particularly meaningful to describe the behavior of the system near its critical point.

▷ **1-1** Show that the magnetization near the critical point behaves as $m \simeq (T - T_c)^\beta$, where β is a critical exponent.

▷ **1-2** Investigate under the same conditions, the behavior of the magnetic susceptibility and that of the specific heat. Find also the behavior of the order parameter at the critical point as a function of the magnetic field.

▷ **1-3** We now consider a space dependent magnetization. Generalize the Landau free energy to take into account the energy cost of having a space dependent magnetization. Derive from this the behavior of the magnetic suspectibility as function of wave vector then as function of space. Calculate the critical exponent ν.

▷ **1-4** Show that the hyperscaling relation $\gamma + 2\beta = d\nu$ only holds for a specific dimension. What is meaning of "universality class" in this context ? Why are the values of the critical exponents which are measured in real systems often different from the values calculated with the Landau model ?

2 Finite size scaling near a continuous phase transition

▷ **2-1** Explain why in practice the divergences of some of the quantities introduced above for an infinite system can not be observed in practice in a simulation. What is the behavior of the correlation length for a system of size L at the critical point ?

▷ **2-2** Within a scaling approach, justify the following relation :

$$\frac{C_L(0)}{C(t)} = F\left(\frac{\xi_L(0)}{\xi_\infty(t)}\right),$$

where F is a scaling function, $C_L(0)$ is the value of the specific heat at the critical point for a system of size L.

▷ **2-3** Assuming that the ratio $\frac{\xi_L(0)}{\xi(t)}$ is finite and independent of L, deduce that $t \simeq L^x$ where x is an exponent to be determined.

▷ **2-4** Show that $C_L(0) \simeq L^y$, where y is to be determined.

▷ **2-5** Using the same method, show that the magnetic susceptibility $\chi \simeq L^z$.