M2 PTSC et MSA
2014-2015

Simulation numérique en physique statistique
TD n° 8

Totally asymmetric exclusion model

Références

[1] An exact solution of a 1D asymmetric exclusion model with open boundaries, B. Derrida, E. Domany and D. Mukamel, J. Stat. Phys., 69, 667 (1992)

[2] Finite-size effects and shock fluctuations in the simple exclusion process, S. Janowsky and J. Lebowitz, Phys. Rev. A, 45, 618 (1992)

1 Asymmetric exclusion model with open boundaries [1]

We first consider a 1D closed system of N sites, ruled by a master equation.

▷ **1-1** Write the conditions of detailed balance and stationarity. To show that they are different conditions, discuss the case of a biased random walker evolving on a ring of N sites.

We now consider an open system where each particle on a given site can move with rate 1 to the site to its right if empty. At boundaries, particles are injected into site 1 with rate α and may leave the system at site N with rate β.

▷ **1-2** Write the expression of the current between the site i and $i+1$. Simplify this using a mean field approximation, and deduce from this recursion relations satisfied by the average occupation number t_i at site i.

▷ **1-3** Solve these recursions using a graphical method. Analyze each possible phases which the system can display in terms of α and β, their corresponding densities and average current.

2 Shock fluctuations in the asymmetric exclusion process [2]

We now consider particles on a lattice of N sites on a circle evolving according to asymmetric exclusion process. We study the effect of a blockage site located at site L, where particles can hop with rate r as opposed to rate 1 on normal sites.

▷ **2-1** Using mean-field, derive the density before and after the blockage site

▷ **2-2** Under which conditions the density stays uniform and does not present a shock ?